Week 7 notes

$$(\varphi, \pi) := \int \varphi(x)\, \pi(x)\, dx.$$

$$\pi(x) = \frac{\gamma(x)}{Z}, \qquad Z = \int \gamma(x)\, dx.$$

$X_i \sim \pi$ (direct / rejection)

$$(\varphi, \pi) := \int \varphi(x)\, \underbrace{\pi(x)\, dx} \approx \int \varphi(x)\, \underbrace{\pi^N(dx)}$$

$$\pi^N(dx) = \frac{1}{N} \sum_{i=1}^{N} \delta_{X_i}(dx), \qquad \varphi(y) = \int \delta_y(dx)\, \varphi(x)$$

$$:= \underbrace{\frac{1}{N} \sum_{i=1}^{N} \varphi(X_i)}.$$

① Unbiased:

$$\mathbb{E}(\cdot) = \frac{1}{N} \sum_{i=1}^{N} \underbrace{\mathbb{E}[\varphi(X_i)]} = (\varphi, \pi) \checkmark$$

② variance

$$\text{variance}(\cdot) = \frac{\mathrm{var}_\pi(\varphi)}{N}.$$

$$\mathbb{P}(X > c) = 10^{-10}$$

$$\mathbb{P}(X > c) = \mathbb{E}[\mathbb{1}_{\{X > c\}}] \approx \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}_{\{X_i > c\}}$$

$$\pi(x) \leq M q(x)$$

$$(\varphi, \pi) := \frac{\int \varphi(x)\,\gamma(x)\,dx}{\int \gamma(x)\,dx}$$

MCMC: $K(x_n \mid x_{n-1})$

$$X_n = a X_{n-1} + \sigma W_n \quad \Rightarrow \quad K(x_n \mid x_{n-1})$$

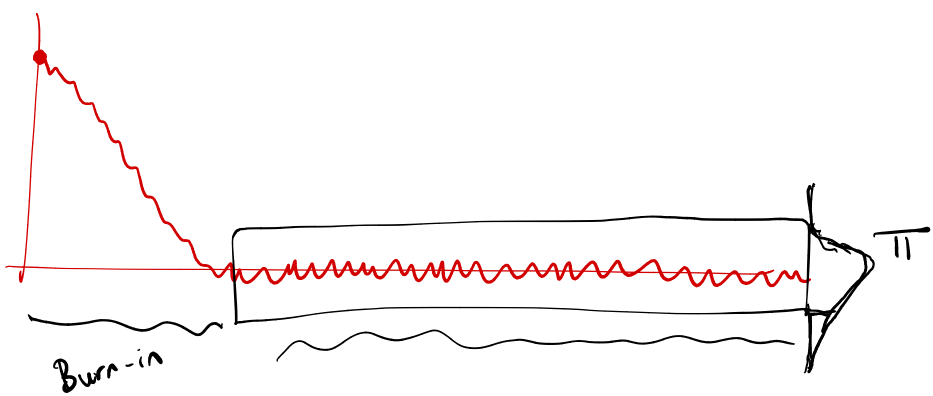
$$W_n \sim \mathcal{N}(0,1) \qquad = \mathcal{N}(x_n; a x_{n-1}, \sigma^2)$$

$$K(\cdot \mid x_{n-1}), \qquad K(x_n \mid \cdot)$$

$$K(X_{n-1}, \cdot) \qquad K(x_{n-1}, dx_n)$$

$$X_0 \xrightarrow{K} X_1 \xrightarrow{K} X_2 \xrightarrow{K} X_3 \to \cdots \to X_n \xrightarrow{n \to \infty}$$

$$P_0 \quad P_1 \quad P_2 \quad P_3 \quad \cdots \quad P_n \to P_\infty$$



$$\boxed{\pi(x) = \int K(x|y)\,\pi(y)\,dy} \longleftrightarrow K \text{ being } \pi\text{-inv.}$$

$$\Uparrow$$

$$\boxed{\pi(x)\,K(x'|x) = \pi(x')\,K(x|x')} \to \text{sufficient} \to \text{detailed balance}$$

$$\Rightarrow \int \pi(x)\,K(x'|x)\,dx' = \int \pi(x')\,K(x|x')\,dx'$$

$$= \pi(x) = \int \pi(x')\,K(x|x')\,dx'$$

$$\boxed{u \leq r(x, x')} \qquad \boxed{\log u \leq \log r}$$

IS code example

$$\varphi^N_{SNIS} = \sum_{i=1}^{N} w_i\,\varphi(X_i), \qquad w_i = \frac{W_i}{\sum_{j=1}^{N} W_j} \qquad W_i = \frac{\gamma(X_i)}{q(X_i)}$$

$p(y|X_i)\pi(X_i)$

$\log W_i$

$$\to w_i = \frac{\exp(\log W_i)}{\sum_{j=1}^{N} \exp(\log W_j)}$$

$$\widetilde{\log W_i} = \log W_i + c$$

$$w_i = \frac{\exp(\widetilde{\log W_i})}{\sum \cdots} = \frac{\exp(c)\exp(\log W_i)}{\exp(c)\sum_{j=1}^{N} \exp(\log W_j)}$$